



# Quantum Versus Classical Entanglement: Eliminating the Issue of Quantum Nonlocality

Andrei Khrennikov<sup>1</sup>

Received: 18 October 2019 / Accepted: 4 January 2020 / Published online: 21 January 2020  
© The Author(s) 2020

## Abstract

We analyze the interrelation of quantum and classical entanglement. The latter notion is widely used in classical optic simulation of some quantum-like features of light. We criticize the common interpretation that “quantum nonlocality” is the basic factor differing quantum and classical realizations of entanglement. Instead, we point to the breakthrough Grangier et al. experiment on coincidence detection which was done in 1986 and played the crucial role in rejection of (semi-)classical field models in favor of quantum mechanics. Classical entanglement sources produce light beams with the coefficient of second order coherence  $g^{(2)}(0) \geq 1$ . This feature of classical entanglement is obscured by using intensities of signals in different channels, instead of counting clicks of photo-detectors. The interplay between intensity and clicks counting is not just a technicality. We elevate this issue to the high foundational level.

## 1 Introduction

The classical electromagnetic field can be successfully used to model some basic features of genuine quantum physical systems (see, e.g., [1–3] and recent review [4] and the references herein). In particular, classical field modeling is a helpful tool for simulation in quantum information theory. However, the foundational output of quantum-like modeling with classical light and other types of waves is not straightforward. In this note, I would like to discuss the foundational meaning of so-called “classical entanglement” and the widely spread view that “quantum nonlocality” (whatever it means) plays the crucial role in distinguishing classical and genuine quantum entanglements. The “quantum nonlocality” viewpoint was clearly formulated by Spreeuw in widely cited paper [2] which is often mentioned by experimenters for foundational justification of their activity:

---

✉ Andrei Khrennikov  
Andrei.Khrennikov@lnu.se

<sup>1</sup> International Center for Mathematical Modeling in Physics and Cognitive Sciences, Linnaeus University, Växjö 351 95, Sweden

It is found that the model system (*classical electromagnetic field*) can successfully simulate most features of entanglement, but fails to simulate quantum nonlocality. Investigations of how far the classical simulation can be pushed show that quantum nonlocality is the essential ingredient of a quantum computer, even more so than entanglement. The well known problem of exponential resources required for a classical simulation of a quantum computer, is also linked to the nonlocal nature of entanglement, rather than to the nonfactorizability of the state vector.

And then he pointed out that

However, the (*classical-quantum*) analogy fails to produce effects of quantum nonlocality, thus signaling a profound difference between two types of entanglement: (i) “true,” multiparticle entanglement and (ii) a weaker form of entanglement between different degrees of freedom of a single particle. Although these two types look deceptively similar in many respects, only type (i) can yield nonlocal correlations. Only the type (ii) entanglement has a classical analogy.

[Comments in italic were added by the author of the present paper.]

However, one can proceed without referring to mysterious quantum nonlocality—by taking into account that quantum theory is about acts of observations. These acts are characterized by *individuality and discreteness*. This crucial point in understanding of quantum theory was missed by authors discussing classical entanglement. They missed that the main deviation of classical light models from quantum theory is not only in the states, but in descriptions of measurement procedures.<sup>1</sup> The classical and semiclassical descriptions of measurements are based on intensities of signals in different channels. The quantum description of measurements is based on counting the discrete events, clicks of detectors, with the aid of the Born’s rule. Operating with intensities obscures the problem of *coincidence detection*. We recall that quantum theory predicts that the relative probability of coincidence detection given by the coefficient of second order coherence  $g^{(2)}(0)$  is zero (for one photon states), but in (semi-)classical models  $g^{(2)}(0) \geq 1$ .

*Genuine quantum theory differs from classical light models reproducing quantum correlations not by “quantum nonlocality”, but by the magnitude of second order coherence.* Classical and semiclassical models were rejected long ago as the result of Grangier et al. [7] experiment on coincidence detection (see [8] for historical review on such experiments).

In fact, the main issue is the difference between classical and quantum superpositions, not entanglements. Our message is that each state-superposition has to be endowed with a proper measurement procedure. Superposition endowed with the classical measurement procedure crucially differs from superposition endowed

<sup>1</sup> As a comment on the first version [5] of this paper, I received email from Gerd Leuchs with the reference to recent review [6]. Authors of this review do not refer to quantum nonlocality to distinguish classical and “true quantum entanglement”. Their position is closer to my own. We shall discuss it in more detail in Sect. 12.

with the quantum measurement procedure. This difference is elevated to the level of entangled states, classical vs. quantum, without measurement procedures they are also just “things-in-themselves” and as such they are not interesting for physics.

Finalizing the introduction, we stress that “quantum nonlocality” is really misleading notion. As shown in [9], the Bell tests can be consistently interpreted in the purely quantum theoretical framework (without any coupling to Bell’s hidden variables theory [10–14]) as statistical tests of local incompatibility of quantum observables, i.e., as tests of the most fundamental principle of quantum mechanics, *the complementarity principle* [15] (see also [14, 16, 17]). For reader’s convenience, the compact presentation of the “Bohr against Bell argument” is given in Sect. 11.

## 2 Nonlocality Mess

Nowadays playing with the notion “quantum nonlocality” is the real mess. People widely operate with this notion and often without any specification on its meaning. We briefly recall the history of its appearance.

The starting point of propagating of quantum nonlocality through the quantum community was the EPR-paper [18]. The EPR reasoning leading them to the conclusion on incompleteness of quantum mechanics is based on the locality assumption:

LOC Since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system.

In principle, nonlocality (as violation of this assumption) can be counted as a possible alternative to incompleteness of quantum mechanics.<sup>2</sup> This reasoning can lead to the idea on *mystical spooky action at a distance*. This notion is often associated with the EPR-paper [18]. But, for the first time Einstein used this phrase only in 1947 letter to Max Born [19]. (As remarked Boughn [20]: “I wish he hadn’t!” And I agree with him.) It is also important to point out that typically the locality assumption is cited as above. However, it contains the second sentence that is always excluded from citations (Why?); the complete formulation of the locality assumption in the original EPR-paper is as follows [18]:

LOC’ Since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system. This is, of course, merely a statement of what is meant by the absence of an interaction between the two systems.

In the light of the last sentence, it is difficult to couple this assumption with any kind of a spooky action at a distance. If the locality assumption is violated, this simply indicates the presence of some physical interaction between systems. (See “Appendix 1”, Bohmian nonlocality).

<sup>2</sup> See the monograph of Jaeger [14] for detailed analysis tis issue.

We emphasize that Bohr replied to Einstein [21] by pointing that EPR's criterion of physical reality becomes ambiguous in quantum physics (see [14, 22] for details of this debate). Bohr did not question the locality assumption; he did not try to use nonlocality as an alternative to incompleteness of quantum mechanics.

Einstein and Bohr did not understand each other, because they behaved towards quantum mechanics in the totally different ways. For Bohr, quantum mechanics is an observational theory, it is about measurements performed by classical measurement apparatuses on microsystems. In modern terminology, quantum mechanics is an epistemic theory [24]; it is about extraction of knowledge about nature with the aid of measurements. For Einstein, quantum mechanics (as any physical theory) was a descriptive theory providing consistent and complete description of nature. Philosophers also use the notion of ontic theory, i.e., theory describing nature as it is - when nobody looks at it (see also [25]).

For me, the root of disagreement between Einstein and Bohr can be found already in the interpretations of measurement on a single system. (Consideration of compound systems and the EPR-states had just strengthen this disagreement.) For Bohr, quantum mechanics generates predictions on outputs of interaction of a quantum system and a measurement apparatus; for Einstein, quantum mechanics (as any "good physical theory") should generate prediction of "real physical properties of a system" (see also Sect. 6).

In any event, Einstein's message on the possibility a spooky action at a distance approached and excited the quantum community. And at the same time, the seeding issue of (in)completeness of quantum mechanics was totally forgotten.<sup>3</sup> Only philosophers continue to debate the EPR-paper [18] from the completeness-incompleteness viewpoint, see, e.g., [22] for the most recent analysis of this issue.

By criticizing the interpretational output of extended research on classical entanglement, I only criticize coupling to mystical quantum nonlocality. As shown in [9] (see also [20, 26–34], and Sect. 11), quantum mechanics by itself has no coupling to such kind of nonlocality. (This statement is also strongly supported by quantum field theory, e.g., [35, 36].) At the same time, a subquantum theory can in principle be nonlocal, as Bohmian mechanics and other theories with hidden variables considered by Bell [10–12]. However, generally, in spite of the Bell theorem, a subquantum theory can be free of nonlocality of a spooky action at a distance type; see [37, 38] and "Appendix 2" for *prequantum classical statistical field theory* (PCSFT). The latter is the classical random field model beyond quantum theory. (Coupling between PCSFT and quantum mechanics is not so straightforward as in the Bell framework [10–12]). PCSFT pretends [37, 38] that genuine quantum systems can be mathematically represented by classical random fields. So, it is not a part the classical entanglement project. It is a part of extensive studies on classical probabilistic reconstruction of quantum theory (see, e.g., [39–49]).

Thus I also contributed to random field modeling of quantum correlations. Therefore generally I am sympathetic to the classical entanglement project. Moreover, by

---

<sup>3</sup> And in the EPR-paper, reasoning on this issue was heavily based on the Heisenberg uncertainty principle. The latter is a special exhibition of the complementarity principle.

reading Spreeuw's paper [2], I had the impression that, in fact, by writing about "quantum nonlocality" he had in mind the correlations of spatially separated signals, as, e.g., in radio-physics (cf. with the above discussion on quantum nonlocality mess).

### 3 Inter-Intra System Versus Quantum-Classical Entanglements

As stated in recent review [4], "...the name classical entanglement denotes the occurrence of some mathematical and physical aspects of quantum entanglement in classical beams of light. ... the term 'classical' in the name classical entanglement, indicates the nonquantum nature of the excitation of the electromagnetic field. ... A typical example thereof is given by a collimated optical beam with non-uniform polarization pattern." We continue by citing [2]: "It should be noted that the choice of optical waves is not essential for the analogy. Other classical waves such as sound, water waves, or even coupled pendula could be used in principle."

In short, classical entanglement is associated with the "nonquantum nature of the excitation of the electromagnetic field".<sup>4</sup> This paper is directed against the statement that the difference between classical and genuine quantum entanglements is due to quantum nonlocality. We stress that comparison classical-quantum entanglements is typically coupled to comparison intra-inter system entanglements. Intra-entanglement is between degrees of freedom of a single system and inter-entanglement is between the degrees of freedom of two subsystems,  $S_1$  and  $S_2$  of compound system  $S = (S_1, S_2)$ .

This is the good place to make the remark on the relationship between single-particle theory, many-particle theory, quantum field theory, and classical field theory. I think that the root of misunderstanding in comparison of classical and quantum entanglements can be found in free operation with notion of "excitation" of a field, classical versus quantum ("nonquantum nature of the excitation of the electromagnetic field"). By comparing classical and quantum entanglements, it is better to appeal to classical and quantum field theories. In the latter, quantum particles(systems) appear as excitation of quantum fields. From this viewpoint, aforementioned excitations of classical electromagnetic field are multiparticle excitations compounded of a huge number of quantum excitations, photons. The degrees of freedom of such multiphoton excitations differ crucially from combination of the degrees of freedom of one (or a few) genuine quantum excitations. And *the origin of the difference is in the multiparticle character of classical excitations*.

As is presented in works on classical entanglement [1–4], its modeling is possible only in the intrasystem context. One may conclude that this feature of entanglement plays the crucial role in distinguishing classical and quantum entanglements. This reasoning also leads to conclusion that only intersystem entanglement is "true

<sup>4</sup> Modes of other classical waves can be used as well. Moreover, classical systems of any origin can be considered, see, e.g., [49] on entanglement of classical Brownian motions.

quantum entanglement” (since intrasystem entanglement can be generated even with classical fields).

In this paper, we demonstrate that classical intrasystem entanglement differs fundamentally not only from quantum intersystem entanglement, but even from quantum intrasystem entanglement. Thus, comparison classical-quantum entanglements has no relation to intra-inter comparison (and, hence, no relation to quantum nonlocality).

This comparative analysis of inter-intra system versus quantum-classical entanglements can be completed by the following remark. The impossibility of classical representation of intersystem entanglement is related only to the very spacial class of the field models elaborated in the classical entanglement project [4] (cf. [37, 38]: in PCSFT, both types of entanglement (intra and inter) can be realized, but they have different mathematical representations, see "Appendix 2" for further discussion).

Finally, we note that the intra-inter system difference of entanglements is invisible in the quantum theoretical framework. In particular, this difference cannot be justified with the aid of the Bell type inequalities (see [9] and Sect. 11). To distinguish intra-intersystem entanglements, we have to go beyond quantum theory (see [37, 38] and "Appendix 2").

#### 4 Grangier et al. Experiment Separating Classical Field Theories from Quantum Mechanics

We start with citing the breakthrough paper of Grangier et al. [7]:

However, there has still been no test of the conceptually very simple situation dealing with single-photon states of the light impinging on a beam splitter. In this case, quantum mechanics predicts a perfect anticorrelation for photodetections on both sides of the beam splitter (a single-photon can only be detected once!), while any description involving classical fields would predict some amount of coincidences.

Following [7], denote by  $p_1, p_2$  the probabilities of detection in two channels after beam splitter and by  $p_c$  the coincidence probability. Then by using the semiclassical model of detection it is easy to show that

$$p_c \geq p_1 p_2. \quad (1)$$

This inequality means clearly that the classical coincidence probability  $p_c$  is always greater than the accidental coincidence probability, which is equal to  $p_1 p_2$ . The violation of inequality (1) thus gives an anticorrelation criterion, for characterizing a nonclassical behaviour of light (see [7, 8]).

The crucial theoretical point is that in classical and semiclassical models the basic physical quantity is intensity of a signal. In Grangier et al. experiment, these are  $I(t)$ , intensity of imprinting on the beam splitter, and  $I_1(t), I_2(t)$  are intensities of signals in the two output channels. The use of intensities, instead of counting of clicks, obscures the coincidence detection problem. We claim that this is not just a

technicality, but the very important foundational issue. And we shall continue discussion in following sections. However, the reader who is not so much interested in foundational questions can jump directly to conclusion (Sect. 12). The main critical point has already been presented.

Finally, we remark PCSFT [37, 38] suffers of the same problem as the classical entanglement models—the “double detection loophole” (see “Appendix 3” for further discussion and attempts to close this loophole by using the threshold detection scheme).

## 5 Quantum Measurements

Consider a quantum observable  $A$  represented by Hermitian operator  $\hat{A}$  with purely discrete spectrum ( $a_i$ ). By the spectral postulate of quantum mechanics any measurement of  $A$  produces one of the values  $a_i$  (as the result of interaction of a quantum system with an apparatus used for  $A$ -measurement). Thus quantum measurements are characterized by individuality of outputs. This crucial feature of quantum measurements was emphasized by Bohr who invented the notion of *phenomenon* [15] (see also [16, 17]):

... in actual experiments, all observations are expressed by unambiguous statements referring, for instance, to the registration of the point at which an electron arrives at a photographic plate. ... the appropriate physical interpretation of the symbolic quantum mechanical formalism amounts only to predictions, of determinate or statistical character, pertaining to individual phenomena ... . ([15], v. 2, p. 64)

Thus, although quantum theory produces statistical predictions, its observables generate individual phenomena. Discreteness of detection events is the fundamental feature of quantum physics justifying existence of quantum systems, carriers of quanta. It is commonly accepted that axiomatic of quantum theory does not contain the special postulate on discrete clicks and the statistical interpretation of probabilities.<sup>5</sup>

One may point to the existence of quantum observables with continuous spectra. The problem of their measurement was analyzed in detail by von Neumann [50]. His analysis implies that measurement of an observable with continuous spectrum has to be reduced to measurements of observables with discrete spectra approximating it. This is the complex foundational issue and we would not go into a deeper discussion; our considerations are restricted to observables with discrete spectra.

<sup>5</sup> However, as was pointed by Plotnitsky (unpublished paper), “it depends on what he sees as axiomatic of quantum theory. The structure of complex Hilbert space does not. But once one introduces projectors and the Born’s rule to relate quantum state to the outcome of experiment, both discreteness and probability enter. The Born rule is not part of the Hilbert space structure and, while mathematically natural (in connections the complex quantities of the formalism to real one and the probability) it is brought ad hoc, and why it works, and it works perfectly, is enigmatic. In a way—that is the main quantum mystery—why Born’s rule works.”

In the classical wave framework the origin of the analog of the quantum Born's rule, so to say the Born's rule for intensities is straightforward. If a classical wave has two orthogonal components, i.e.,

$$\Phi(x) = \Phi_1(x) + \Phi_2(x), \quad (2)$$

with intensities  $I_1$  and  $I_2$ , then corresponding probabilities can be expressed in the form  $p_j = I_j/(I_1 + I_2), j = 1, 2$ , and intensities are given by the “classical Born's rule”:

$$I_j = \|\Phi_j\|_{L_2}^2 = \int |\Phi_j(x)|^2 dx. \quad (3)$$

However, this is the separate question whether the coefficients  $p_j$  can really be interpreted as probabilities of discrete events.

In papers on classical entanglement, there are considered expansions of state-vectors with respect to orthonormal bases in complex Hilbert spaces.<sup>6</sup> Such expansions may make the impression that the standard quantum mechanical scheme of measurement can be applied. This is not the case. For classical signals, it is impossible to project the initial state on the state corresponding to one concrete outcome. In the two slit experiment with classical waves, a wave propagating from the source passes both slits at the same time.

Finally, we remark that in classical field theory the method of complex Hilbert space started to be used even before appearance of quantum mechanics. We can mention, for example, *Riemann-Silberstein representation*,  $\Psi(x) = E(x) + iH(x)$ , for the classical electromagnetic field. In this representation, the Maxwell equation has the form of the Schrödinger equation. So, studies on classical entanglement are consistent with complex Hilbert space analysis of classical signals.

## 6 Reality Without Realism (RWR) Interpretation of Quantum Mechanics

The above discussion on quantum discreteness matches perfectly the RWR interpretation of quantum mechanics that was elaborated in a series of works of Plotnitsky (see [22] and references herein; see also [23] for out joint paper). This is one of the versions of the Copenhagen interpretation<sup>7</sup>. I now present RWR. (This is my interpretation of RWR. It may differ from Plotnitsky's own views.)

We start by remarking that often Bohr's views are presented as idealism. But, this is misunderstanding. He definitely did not deny reality of *quantum systems*, say electrons or atoms. However, as pointed out in Sect. 5, quantum mechanics does not

<sup>6</sup> The tensor product structure is typically emphasized. But this is not the main issue, see Sect. 7 on classical vs. quantum interpretation of states' superposition.

<sup>7</sup> Bohr had never formulated the Copenhagen interpretation exactly. The quantum community uses a variety of interpretations pretending to express Bohr's views. Therefore, Plotnitsky proposed to speak about interpretations in the spirit of Copenhagen. RWR is one of such interpretations.



describe genuine physical properties of quantum systems. Bohr stressed that measurement's output cannot be disassociated from a measurement apparatus and generally the complex of experimental physical conditions, experimental context. We can point to two common misuses of quantum theory (well, from the RWR-viewpoint). On one hand, one may neglect the role of experimental context and try to assign measurement's output directly to a quantum system. We call this approach "naive realism". From the Bohr-Heisenberg viewpoint, this approach should be rejected as contradicting the Heisenberg uncertainty relation and generally the complementarity principle. Another misinterpretation is forgetting about the existence of quantum systems (the reality counterpart of RWR). Following Bohr, electron exists! And the output of measurement is assigned to this concrete electron (prepared for measurement), but, of course, the WR-counterpart of Bohr-Plotnitsky interpretation has also to be taken into account. Therefore quantum theory is about such individual assigning of outputs (quantum phenomena). This is the origin of discreteness of quantum measurements. That is why measurement of intensity of a beam of classical light is not a quantum phenomenon. As was found by realization of the classical entanglement project, generally the WR-counterpart of RWR should be taken into account even for classical light. But, surprisingly, the R-counterpart cannot be applied. Hence, classical optics measurements do not produce quantum phenomena in Bohr's meaning.

Finally, we remark that measurements for a quantum system in the intra-entangled state satisfies both the R- and WR-counterparts of of WRW; so, their outputs are quantum phenomena.

## 7 Comparing Classical and Quantum Superpositions

From my viewpoint, the misleading journey towards classical entanglement starts already with identifying classical and quantum superpositions. Physically these superpositions are totally different, in spite of the possibility to represent them by the same mathematical expression.

Consider a classical electromagnetic field with  $n$  orthogonal modes corresponding to frequencies  $(\nu_j, j = 1, \dots, n)$  with complex amplitudes  $(C_j)$ . This field can be represented in  $n$ -dimensional complex Hilbert space with the basis  $(e_j \equiv |\nu_j\rangle, j = 1, \dots, n)$  :

$$\Psi = \sum_j C_j |\nu_j\rangle. \quad (4)$$

This vector can be normalized:

$$|\psi\rangle = \sum_j c_j |\nu_j\rangle, \quad (5)$$

where  $c_j = C_j / \sqrt{\sum_j |C_j|^2}$ .

*What is the main difference of classical field superposition (5) from the genuine quantum superposition?*

The main difference is in measurement procedures determining probabilities  $p_j = |c_j|^2$ . For the classical field, it is impossible to detect discrete clicks in  $n$  channels without coincidence detections, where the degree of coincidence is determined by coefficient  $g^{(2)}(0)$ . Thus, to see the difference between classical and quantum light, one need not consider formal entanglement expressions corresponding for different degrees of freedom. It is sufficient to consider one degree of freedom and superposition.

The first detailed comparison of classical and quantum superpositions was presented in Dirac's book [51]. At the very beginning of quantum physics, Dirac pointed out ([51], p.14.):

*It is important to remember, however, that the superposition that occurs in quantum mechanics is of an essentially different nature from any occurring in the classical theory,...*

Unfortunately, this message was forgotten. What is this essential difference? So, we follow Dirac and his reasoning is rather long.<sup>8</sup>

“The nonclassical nature of the superposition process is brought out clearly if we consider the superposition of two states  $A$  and  $B$ , such that there exists an observation which, when made on the system in state  $A$ , is certain to lead to one particular result,  $a$  say, and when made on the system in state  $B$ , is is certain to lead to some different result,  $b$  say. What will be the result of the observation when made on the system in the superposition state? The answer is that the result will be sometimes  $a$  and sometimes  $b$ , depending on the relative weights of  $A$  and  $B$ .” ([51], p.13.)

Here Dirac clearly point to “one particular result” of observation. This particularity of observation's result is the essence of the quantum superposition.

## 8 Comparing Classical and Quantum Entanglements

Following papers on classical entanglement, consider two degrees of freedom of the classical electromagnetic field which can be jointly measured. The four dimensional complex Hilbert space contains states of this field that are nonseparable and formally they can be treated as entangled. Here “entangled” is understood purely mathematically, as the special form of representation in complex Hilbert space endowed with the tensor product structure. As in the case of superposition, the devil is not in the state, but in measurement. For the classical electromagnetic field, photo-detectors

<sup>8</sup> And we have to follow this reasoning. In book [52], its author (who definitely understands well the difference between two types of superposition) presented completion of the above citation of Dirac: “... as is shown by the fact that the quantum superposition principle demands indeterminacy of observations in order to be capable of a sensitive physical interpretation.” ([51], p. 14.) Here we find the emphasis on observation. However, “indeterminacy of observation” does not specify observations' discrete character, its ability to generate physical phenomena, say in the form of clicks of detectors.

cannot produce phenomena, in Bohr's sense. The measurement procedure suffers from coincidence detection.

## 9 Quantum Information: Role of Discrete Clicks of Detectors

The bit is a portmanteau of binary digit and this preassumes the discrete structure of information represented by bits. Qubit is commonly introduced by “replacing the classical idea of a binary digit with a quantum two state system.” (see Schumacher [53]). This definition reflects common neglect by the role of measurement. Unfortunately, in quantum information research one typically operates with states forgetting about extracting information from them (see Jaeger's monograph [52] for further discussion). As we have seen in Sect. 8, two states superpositions by their selves are not quantum. Genuine quantum superposition is combination of a state and measurement procedure extracting discrete alternatives. Thus, quantum information theory is not reduced to linear algebra in complex Hilbert space. Its main component is quantum measurement procedure. The main value of quantum information (as well as classical one) is in the possibility to extract from states discrete events, e.g., clicks of a photo-detector or dots on a photo-emulsion screen.

## 10 Has Classical Entanglement Anything to Do with Original Bell Argument?

The above critique of attempts to couple studies on classical entanglement with quantum theory can also be applied to attempts to couple classical random field correlations violating Bell type inequalities with the original Bell argument [10–12]. Bell applied classical probability theory to derive his inequality. The later was used to compare the classical probabilistic representation of correlations with the quantum theoretical description.

We recall that in classical probability theory observables are represented by random variables, functions on sample space. Denote the latter by symbol  $\Lambda$  (although mathematicians typically use symbol  $\Omega$ ). Then a random variable  $\xi : \Lambda \rightarrow \mathbf{R}$  and by definition of a function it takes only one value  $\xi(\lambda)$  for each  $\lambda \in \Lambda$ . Thus by getting the clicks in both channels one understand that it is impossible to represent such measurements by classical random variables.

We remark that originally (following EPR-paper [18]) Bell was interested in explanation of perfect correlations. In his original inequality [10], it was assumed that ranges of values of quantum and classical observables should coincide, i.e., the range of values is the two point set  $\{\pm 1\}$ . A classical random variable is a function  $\xi : \Lambda \rightarrow \{\pm 1\}$ . And if one would accept that for some set of  $\lambda$ s,  $\xi$  is multivalued, i.e., at the same time  $\xi(\lambda) = -1$  and  $\xi(\lambda) = +1$ , then classical probability theory stops to work. There is no way to derive the Bell inequality. In the CHSH-framework, the range of values of observables was extended to the segment  $[1, +1]$ . However, this was done with only one purpose, namely, to include value 0 corresponding to

non-detection event. Thus in real physical modeling the range of values is given by the discrete set  $\{-1, 0, +1\}$ .

Moreover, as was already pointed out, von Neumann emphasized [50] that any Hermitian operator  $\hat{A}$  with continuous spectrum is just a symbolic expression of converging sequence of quantum observables with discrete spectra, representing approximate measurements.

The above remarks on discreteness of quantum and classical observables were presented only to underline the astonishing difference between measurement procedures in the Bell framework and in classical optics. Even classical random variables with continuous range of values are mathematically represented by single-valued functions.

### 10.1 “Superstrong Quantum Correlations”: Comparing Original Bell Inequality and CHSH-Inequality

Excitement of researchers violating the CHSH inequality (theoretically or experimentally) with classical field correlations is well understandable. The statement on “superstrong quantum correlations” that cannot be represented as classical correlations has been emphasized in the quantum community. Typically correlations were associated with states and the issue of quantum vs. classical measurement procedures was practically ignored.

This is the good place to point that transition from the original Bell inequality [10] to the CHSH-inequality [13] was not so innocent from the foundational viewpoint. The original Bell inequality is about explicit correlations and hence comparison of the concrete values of observables (cf. [18]). It is evident that, for this inequality, transition from discrete clicks to intensities is nonsense. In the CHSH-framework, this basic issue was obscured. Instead, the issue of “superstrong quantum correlations” was elevated (see [54] for discussion; see also [55] for related theoretical study). Nowadays we are much closer to performance of experiments on violation of the original Bell inequality (see [54] for analysis of the present situation in theory and experiment). Such experiments will immediately distance quantum physics from its classical simulation.

## 11 Bell’s Inequalities as Tests of Observables’ Incompatibility

The unconventional interpretation of Bell’s type inequalities was proposed in recent author’s paper [9]. This paper presents purely quantum mechanical treatment of these inequalities, i.e., without any relation to hidden variables. Observables measured in experiments are coupled directly to quantum observables. It was shown that in this framework these *inequalities express the compatibility-incompatibility interplay for local observables*. Thus quantum theory has nothing to do with nonlocality. For reader’s convenience we briefly present the aforementioned analysis.

The quantum theoretical CHSH-correlation function has the form:

$$\langle \mathcal{B} \rangle_\psi = \frac{1}{2} [\langle \hat{A}_1 \hat{B}_1 \rangle_\psi + \langle \hat{A}_1 \hat{B}_2 \rangle_\psi + \langle \hat{A}_2 \hat{B}_1 \rangle_\psi - \langle \hat{A}_2 \hat{B}_2 \rangle_\psi], \quad (6)$$

where  $\psi$  is a pure quantum state (mixed states can be considered as well). (This quantum theoretical correlation functions is compared with the experimental CHSH-correlation function).

In the quantum framework, the CHSH-correlation function can be expressed with the aid of the Bell-operator:

$$\hat{\mathcal{B}} = \frac{1}{2} [\hat{A}_1(\hat{B}_1 + \hat{B}_2) + \hat{A}_2(\hat{B}_1 - \hat{B}_2)] \quad (7)$$

as

$$\langle \mathcal{B} \rangle_\psi = \langle \psi | \hat{\mathcal{B}} | \psi \rangle. \quad (8)$$

By straightforward calculation one can derive the Landau identity:

$$\hat{\mathcal{B}}^2 = I - (1/4)[\hat{A}_1, \hat{A}_2][\hat{B}_1, \hat{B}_2]. \quad (9)$$

This identity implies that if at least one of commutators  $[\hat{A}_1, \hat{A}_2], [\hat{B}_1, \hat{B}_2]$  equals zero, i.e., if at least one pair of observables,  $(A_1, A_2)$  or (and)  $(B_1, B_2)$ , is compatible, then for any state  $\psi$ ,

$$\sup_{\|\psi\|=1} |\langle \mathcal{B} \rangle_\psi| = \|\hat{\mathcal{B}}\| = 1, \quad (10)$$

i.e., for each state  $\psi$ ,

$$|\langle \mathcal{B} \rangle_\psi| = \|\hat{\mathcal{B}}\| \leq 1. \quad (11)$$

This is the quantum version of the CHSH-inequality. The classical bound by 1 has the purely quantum explanation.

Simple spectral analysis shows (see ) that if the product of commutators is not equal to zero, i.e., in both pairs  $(A_1, A_2)$  and  $(B_1, B_2)$  of observables are incompatible, then either

$$\sup_{\|\psi\|=1} |\langle \mathcal{B} \rangle_\psi| = \|\hat{\mathcal{B}}\| > 1, \quad (12)$$

or, for  $\hat{\mathcal{B}}_- = \frac{1}{2}[\hat{A}_1(\hat{B}_2 - \hat{B}_1) + \hat{A}_2(\hat{B}_1 - \hat{B}_2)]$ ,

$$\sup_{\|\psi\|=1} |\langle \mathcal{B}_- \rangle_\psi| = \|\hat{\mathcal{B}}_-\| > 1 \quad (13)$$

This condition can be rewritten in a compact form. Denote by  $\sigma$  some permutation of the indexes for the  $A$ -observables and the indexes for the  $B$ -observables and denote by  $\hat{\mathcal{B}}_\sigma$  the operator with corresponding permutation of indexes. If the product of commutators is not equal to zero, then

$$\max_{\sigma} \sup_{\|\psi\|=1} |\langle \mathcal{B}_{\sigma} \rangle_{\psi}| > 1, \tag{14}$$

i.e., there exists some state  $\psi$  such that the CHSH-inequality is violated at least for one of correlations  $\langle \mathcal{B}_{\sigma} \rangle_{\psi}$ .

The issue of locality can be formalized by introducing the tensor product structure on the state space  $H$ , i.e.,  $H = H_1 \otimes H_2$  and considering observables represented by Hermitian operators in the form  $\hat{A}_i = \hat{A}_i \otimes I$  and  $\hat{B}_i = I \otimes \hat{B}_i$ , where Hermitian operators  $\hat{A}_i$  and  $\hat{B}_i$  act in spaces  $H_1$  and  $H_2$ , respectively. Then the condition of commutativity respects the tensor product structure, since  $[A_1, A_2] = [\hat{A}_1, \hat{A}_2] \otimes I$  and  $[B_1, B_2] = I \otimes [\hat{B}_1, \hat{B}_2]$ .

Now, if the tensor product structure corresponds to the compound system structure, then  $[\hat{A}_1, \hat{A}_2] \neq 0$  and  $[\hat{B}_1, \hat{B}_2] \neq 0$  are conditions of *local incompatibility of observables*. Thus satisfaction-violation of the CHSH-inequality is completely determined by these local conditions.<sup>9</sup>

*By interpreting the Bell type inequalities as describing the compatibility-incompatibility interplay we cannot point to any difference between “intersystem and intrasystem entanglement”.*

At the same time, analysis presented in the previous sections points to crucial difference between classical and quantum entanglements.<sup>10</sup> For classical light, the presented incompatibility interpretation of the CHSH inequality for quantum systems has to be used with caution. We restrict considerations to intra-entanglement. Of course, the mathematical structure of states and operators is the same. Thus, all above calculations are valid even in the classical entanglement framework. However, the physical meaning of operators is not the same. In quantum physics, the operators represent measurements procedures which do not suffer of the double detection loophole; in classical optics, the same operators represent measurements procedures which suffer of this loophole. It is not clear whether one can extend the complementarity principle to such measurements. (This is the good question to experts in quantum foundations).

PCSFT reproduces quantum correlations without establishing isomorphism of state spaces and physical variables, subquantum→quantum map has a more complex structure. Therefore the above operator analysis of the CHSH-inequality has no direct impact to this theory. To couple consequences of this analysis with PCSFT, we have to understand how complementarity arises through transition from a sub-quantum theory to quantum mechanics.

<sup>9</sup> In the absence of the tensor product structure, we have to impose the constraint that the product of commutators differs from zero. The presence of tensor product structure makes this constraint redundant.

<sup>10</sup> This is the reply to the question of Mario Krenn who told me about extended studies on classical entanglement and asked me to comment them in the light of the purely quantum mechanical analysis of the Bell type inequalities (after my talk at FQMT19, Prague).

## 12 Concluding Remarks

The aim of this note is to distance the technical impact of “classical entanglement” research (both for theory and experiment) [4] from its misleading interpretation, as supporting “quantum nonlocality” [2]. First we present the main points of our analysis of the notion “quantum nonlocality”:

- In modern physics, its using is the real mess.
- The ontic-epistemic (descriptive-observational) viewpoint on scientific theories clarifies misunderstanding between Einstein and Bohr.
- Einstein’s treatment of elements of reality as components of observational theory leads him to really misleading notion of quantum nonlocality, based on a spooky action at a distance.
- Bell type inequalities have the purely quantum interpretation as tests of local incompatibility.

We now list the main conclusions from our analysis of interrelation of classical and quantum entanglements:

- The main issue is the difference between classical and quantum superpositions. It can be explained by Grangier et al. experiment [7, 8].
- The distinguishing feature of quantum measurements is discreteness and individuality of outcomes (as expressed in Bohr’s notion of phenomenon).
- Derivation of quantum(-like) correlations with classical entanglement [2, 4] implies that the Hilbert space formalism has to be distinguished from genuine quantum physics.
- Classical entanglement is not consistent with Bell’s hidden variables theory: coincidence detection blocks the use of random variables.

This comparison of classical and quantum entanglements and critique of the “quantum nonlocality” interpretation of their difference is the main output of the paper.

Finally, we point to the recent review of Korolkova and Leuchs [6] which is the important step towards resolution of the interpretation problems related to interrelation of classical and quantum entanglements. Its authors do not more refer to quantum nonlocality (cf. with previous review [4]). They recognize that the main issue is not the impossibility to generate intersystem entanglement with classical optics. The main issue is the difference between intra system entanglements, classical vs. quantum. And Korolkova and Leuchs, as well as the author of this paper, also emphasize the role of measurement procedures in distinguishing two types of entanglement. They made the following remark on an intra-entangled state of a genuine quantum system: “*This state is the quantum entangled state of type  $|01\rangle + |10\rangle$  a strict correlation of one photon in one arm and no photon in the other or vice versa.*” I would just add that, in fact, the root of the problem lies already in classical vs. quantum interpretation of superposition-state  $|0\rangle + |1\rangle$ .

**Acknowledgements** Open access funding provided by Linnaeus University. This work was supported by the research project of the Faculty of Technology, Linnaeus University, “Modeling of complex hierarchical structures”.

**Open Access** This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article’s Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

## Appendix 1: Subquantum Nonlocality, Bohmian Mechanics

Bohmian mechanics is the most popular example of nonlocal subquantum model. Its nonlocality is in straightforward violation of the locality assumption **LOC'**, as “merely a statement of what is meant by the absence of an interaction between the two systems.” The quantum potential depends nonlocally on coordinates of the sub-systems of a system.

## Appendix 2: Subquantum Modeling of Inter-Intra System Entanglements

One possibility is to appeal to so-called *prequantum classical random field theory* (PCSFT) [37, 38] that is devoted to modeling of *both forms of entanglement*, intra and inter system, with the aid of classical random fields. PCSFT provides the abstract random field representation of quantum averages and correlations. In PCSFT, intra and inter system entanglements have different mathematical representations. The crucial point is that representation of intersystem entanglement (in PCSFT) is impossible without assuming the presence of *a random background field*, a kind of the zero point field (field of vacuum fluctuations) explored in stochastic electrodynamics. In principle, the presence of such a background field can be interpreted as nonlocality, although the use of such a terminology would be really misleading. Say in radiophysics, nobody would associate some mystical features with a random background. However, in this note we shall not present the details of the PCSFT modeling of intra and inter system entanglements. We plan to do this in a future publication.



### Appendix 3: Extracting Discrete Events from Continuous Random Fields

We remark that the Grangier-type experiments were directed against one special model of photo-detection, the semiclassical model (see [56]). “In the semiclassical theory of photoelectric detection, it is found that the conversion of continuous electromagnetic radiation into discrete photoelectrons is a random process.” (see [8]) One can propose other detection models for such conversion. The simplest way to extract discrete events from continuous random fields is to use threshold detection procedures. Such a project was started in the PCSFT-framework [37, 38, 57, 58]. First we consider intrasystem entanglement. In this case, the threshold detection scheme can be designed to exclude the double detection (clicks in both channels) for a dichotomous observable. Here theoretical research was completed by numerical simulation [57]. The coefficient of second order coherence  $g^{(2)}(0)$  is used as the measure “quantumness”; it is possible to construct such classical random fields and the threshold detection scheme that  $g^{(2)}(0) < 1$ .

Now consider intersystem entanglement, in the PCSFT-realization. This realization can also be equipped with a threshold detection scheme and correlations based on probabilities for discrete counts can violate the CHSH-inequality [58]. However, in this case I was not able to close the double detection loophole. The model of (classical field based) intersystem entanglement endowed with the threshold detection scheme is so complex [58] that it is difficult to estimate the magnitude of coefficient  $g^{(2)}(0)$ .

### References

1. Spreuw, R.J.C.: A classical analogy of entanglement. *Found. Phys.* **28**, 361–374 (1998)
2. Spreuw, R.J.C.: Classical wave-optics analogy of quantum information processing. *Phys. Rev. A* **63**, 062302 (2001)
3. Ghose, P., Mukherjee, A.: Entanglement in classical optics. *Rev. Theor. Sci.* **2**, 1–14 (2014)
4. Aiello, A., et al.: Quantum-like nonseparable structures in optical beams. *New J. Phys.* **17**, 043024 (2015)
5. Khrennikov, A.: Quantum versus classical entanglement: eliminating the issue of quantum nonlocality. [arXiv:1909.00267v1](https://arxiv.org/abs/1909.00267v1) [quant-ph]
6. Korolkova, N., Leuchs, G.: Quantum correlations in separable multi-mode states and in classically entangled light. *Rep. Prog. Phys.* **82**, 056001 (2019)
7. Grangier, P., Roger, G., Aspect, A.: Experimental evidence for a photon anticorrelation effect on a beam splitter: a new light on single-photon interferences. *Europhys. Lett.* **1**(4), 173–179 (1986)
8. Thorn, J.J., Neel, M.S., Donato, V.W., Bergreen, G.S., Davies, R.E., Becka, M.: Observing the quantum behavior of light in an undergraduate laboratory. *Am. J. Phys.* **72**, 9! (2004)
9. Khrennikov, A.: Get rid of nonlocality from quantum physics. *Entropy* **21**(8), 806 (2019)
10. Bell, J.S.: On the Einstein Podolsky Rosen paradox. *Physics* **1**, 195–200 (1964)
11. Bell, J.S.: *Speakable and Unsayable in Quantum Mechanics*, Second edn. Cambridge University Press, Cambridge (2004)
12. Bell, J.S.: On the problem of hidden variables in quantum theory. *Rev. Mod. Phys.* **38**, 450 (1966)
13. Clauser, J.F., Horne, M.A., Shimony, A., Holt, R.A.: Proposed experiment to test local hidden-variable theories. *Phys. Rev. Lett.* **23**, 880 (1969)

14. Jaeger, G.: *Quantum Objects: Non-local Correlation, Causality and Objective Indefiniteness in the Quantum World*. Springer, New York, NY (2013)
15. Bohr, N.: *The Philosophical Writings of Niels Bohr*. Ox Bow Press, Woodbridge (1987)
16. Plotnitsky, A.: *Epistemology and Probability: Bohr, Heisenberg, Schrödinger and the Nature of Quantum-Theoretical Thinking*. Springer, Berlin (2009)
17. Plotnitsky, A.: *Niels Bohr and Complementarity: An Introduction*. Springer, Berlin (2012)
18. Einstein, A., Podolsky, B., Rosen, N.: Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.* **47**, 777–780 (1935)
19. Born, M.: *The Born-Einstein Letters 1916–1955*. Macmillan Press, New York (1971)
20. Boughn, S.: There is no action at a distance in quantum mechanics, spooky or otherwise (2018). [arXiv:1806.07925](https://arxiv.org/abs/1806.07925)
21. Bohr, N.: Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.* **48**, 696–702 (1935)
22. Plotnitsky, A.: Spooky predictions at a distance: reality, complementarity and contextuality in quantum theory. *Philos. Trans. R. Soc. A* (2019). <https://doi.org/10.1098/rsta.2019.0089>
23. Plotnitsky, A., Khrennikov, A.: Reality without realism: on the ontological and epistemological architecture of quantum mechanics. *Found. Phys.* **45**(10), 1269–1300 (2015)
24. Atmanspacher, H., Primas, H.: Epistemic and Ontic Quantum Realities. In: Castell, L., Ischebeck, O. (eds.) *Time, quantum and information*, pp. 301–321. Springer, Berlin (2003)
25. Khrennikov, A.: Hertz’s viewpoint on quantum theory. *Activitas Nerv Super* (2019). <https://doi.org/10.1007/s41470-019-00052-1>
26. Accardi, L.: The probabilistic roots of the quantum mechanical paradoxes, In: *The Wave–Particle Dualism. A Tribute to Louis de Broglie on his 90th Birthday*, Diner S., Fargue D., Lochak G., and Selleri F. (eds), D. Reidel Publ. Company: Dordrecht, pp. 47–55 (1984)
27. Khrennikov, A.: *Interpretations of probability; VSP Int. Sc. Publishers, Utrecht/Tokyo, 1999; second edition (completed), De Gruyter, Berlin (2009)*
28. De Muynck, W.: *Foundations of Quantum Mechanics, an Empiricist Approach*. Springer, Dordrecht (2002)
29. Khrennikov, A., Alodjants, A.: Classical (local and contextual) probability model for Bohm-Bell type experiments: no-signaling as independence of random variables. *Entropy* **21**, 157–177 (2019)
30. Boughn, S.: Making sense of Bell’s theorem and quantum nonlocality. *Found. Phys.* **47**, 640–657 (2017)
31. Khrennikov, A.: Bohr against Bell: complementarity versus nonlocality. *Open Phys.* **15**, 734–738 (2017)
32. Kupczynski, M.: Can Einstein with Bohr debate on quantum mechanics be closed? *Philos. Trans. R Soc. A* **375**, 2016039 (2017)
33. Kupczynski, M.: Closing the door on quantum nonlocality. *Entropy* **20**, 877 (2018)
34. Griffiths, R.B.: Quantum nonlocality: myth and reality (2019). [arXiv:1901.07050](https://arxiv.org/abs/1901.07050)
35. Bogoliubov, N.N., Shirkov, N.N.: *Introduction to Theory of Quantized Fields*. Interscience Publishers, Moscow (1959)
36. Haag, R.: *Local Quantum Physics Fields, Particles, Algebras*. Springer, Berlin (1996)
37. Khrennikov, A.: Prequantum classical statistical field theory: Schrödinger dynamics of entangled systems as a classical stochastic process. *Found. Phys.* **41**, 317–329 (2011)
38. Khrennikov, A.: *Beyond Quantum*. Pan Stanford Publication, Singapore (2014)
39. Feynman, R.P.: The Concept of probability in quantum mechanics. In: *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, pp. 533–541. University of California Press (1951)
40. Mackey, G.W.: *Math. Found. Quantum Mech.* W. A. Benjamin Inc., New York (1963)
41. Mancini, S., Man’ko, V.I., Tombesi, P.: Symplectic tomography as classical approach to quantum systems. *Phys. Lett. A* **213**, 1–6 (1996)
42. Man’ko, V.I., Man’ko, O.V.: Spin state tomography. *J. Exp. Theor. Phys.* **85**, 430–434 (1997)
43. Khrennikov, A.: 1999 Interpretations of probability. Utrecht/Tokyo: VSP Int. Sc. Publ.; Berlin: De Gruyter, second edition (2009)
44. Ballentine, L.E.: Interpretations of probability and quantum theory. In: Khrennikov, A.Yu. (ed.) *Foundations of Probability and Physics, Quantum Probability and White Noise Analysis 13*, pp. 71–84. WSP, Singapore (2001)

45. Khrennikov, A.: The principle of complementarity: a contextual probabilistic viewpoint to complementarity, the interference of probabilities, and the incompatibility of variables in quantum mechanics. *Found. Phys.* **35**(10), 1655–1693 (2005)
46. Man'ko, M.A., Man'ko, V.I.: New entropic inequalities and hidden correlations in quantum supratism picture of qudit states. *Entropy* **20**, 692 (2018)
47. Khrennikov, A.: Bell could become the Copernicus of probability. *Open Syst. Inf. Dyn.* **23**, 1650008 (2016)
48. Khrennikov, A., Alodjants, A.: Classical probability model for Bohm-Bell type experiments: no-signaling as independence of random variables. *Entropy* **21**(2), 157 (2019)
49. Allahverdyan, A.E., Khrennikov, A., Nieuwenhuizen, T.M.: Brownian entanglement. *Phys. Rev. A* **72**, 032102 (2005)
50. Von Neumann, J.: *Mathematical Foundations of Quantum Mechanics*. Princeton University Press, Princeton, NJ (1955)
51. Dirac, P.: *The Principles of Quantum Mechanics*, 4th edn. Clarendon Press, Oxford (2012)
52. Jaeger, G.: *Quantum Information. An Overview*. Springer, New York (2007)
53. Schumacher, B.: Quantum coding. *Phys. Rev A* **51**, 2738–2749 (1995)
54. Khrennikov, A., Basieva, I.: Towards experiments to test violation of the original Bell inequality. *Entropy* **20**(4), 280 (2018)
55. Khrennikov, A.Y., Loubenets, E.R.: Evaluating the maximal violation of the original Bell inequality by two-qudit states exhibiting perfect correlations/anticorrelations. *Entropy* **20**(11), 829 (2018)
56. Mandel, L., Wolf, E.: *Optical Coherence and Quantum Optics*. Cambridge University Press, Cambridge (1995)
57. Khrennikov, A., Nilsson, B., Nordebo, S.: On an experimental test of prequantum theory of classical random fields: an estimate from above of the coefficient of second-order coherence. *Int. J. Quantum Inf.* **10**, 1241014 (2012)
58. Khrennikov, A.: Quantum probabilities and violation of CHSH-inequality from classical random signals and threshold type detection scheme. *Prog. Theor. Phys.* **128**, 31–58 (2012)

**Publisher's Note** Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.